$\log_e x$

(3) (a) $\lim_{x\to\infty} \frac{10x^2}{e^x}$ $\frac{\infty}{\infty}$ $\lim_{x\to\infty} \frac{20x}{e^x}$ $\frac{\infty}{\infty}$ $\lim_{x\to\infty} \frac{20}{e^x}$ $\frac{20}{\infty}$ $\boxed{0}$

(3) (b) $\lim_{x\to 1} \frac{\sqrt{x}-1}{\ln x}$ $\frac{0}{0}$ $\frac{\frac{1}{2}x^{-1/2}}{\frac{1}{x}} = \frac{1}{2\sqrt{x}} \cdot \frac{x}{1} = \frac{x}{2\sqrt{x}}$ $\frac{1}{2}$

(3) (c) $\lim_{x\to 0} \frac{x^3-1}{\cos(x)}$ $= \frac{-1}{1}$ $= -1$

Apr 28-7:29 PM

Calculus 120
Unit 5: Odds and Ends

May 30, 2019: Day #2

~~**1. Finish Tests or Review**~~

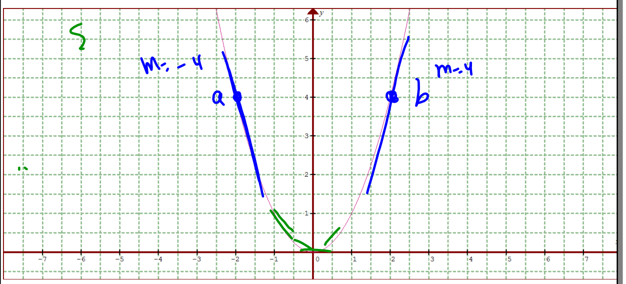
2. Mean Value Theorem

2. Hyperbolic Functions

Jan 9-1:43 PM

Intermediate Value Theorem for Derivatives

If *a* and *b* are any two points in an interval on which *f* is differentiable, then *f* ′ takes on every value between *f* ′*(a)* and *f* ′*(b).*

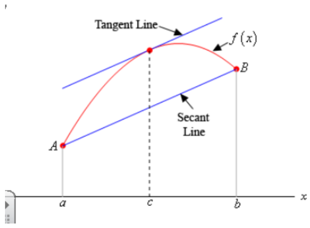


May 29-1:07 PM

Mean Value Theorem

If $y = f(x)$ is continuous at every point on the closed interval [a, b], and differentiable at every point of its interior (a, b), then there is at least one point *c* in (a, b), at which $f'(c) = \frac{f(b)-f(a)}{b-a}$.

In other words, there is a tangent line in the interval that has the same slope as the secant connecting *a* to *b*.



May 29-1:09 PM

Show that the function $f(x) = x^2$ satisfies the hypotheses of the Mean Value Theorem on the interval [0, 2]. Then find a solution *c* to the equation $f'(c) = \frac{f(b)-f(a)}{b-a}$ on this interval.

Sketch a visual.

$f(0) = 0$ $f(2) = 4$

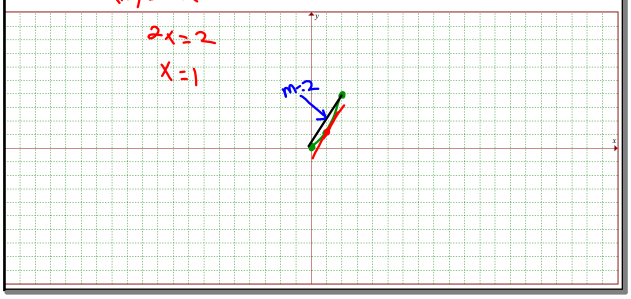
$m = \frac{0-4}{0-2}$

$m = 2$

$f'(x) = 2x$

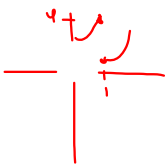
$2x = 2$

$x = 1$



Jun 4-11:52 AM

Explain why the following function fails to satisfy the conditions of the Mean Value Theorem on the interval [-1, 1].



$$f(x) = \begin{cases} x^3+3, & x < 1 \\ x^2+1, & x \geq 1 \end{cases}$$

$f(1) = 1^3+3 = 4$

$f(1) = 1^2+1 = 2$

The function is not continuous @ $x=1$

Jun 4-11:53 AM

Let $f(x)=\sqrt{1-x^2}$, $A=(-1, f(-1))$, and $B=(1, f(1))$. Find a tangent to *f* in the interval (-1, 1) that is parallel to the secant *AB*.

May 29-1:10 PM

Joe left his house at 12:00 and arrived at his destination

200 km away at 2:00. Joe's wife was sleeping for the entire trip. She woke up just as they arrived in town and Joe was driving the 80 km/h speed limit. Joe's wife is a Calculus teacher. How can she use the mean value theorem to prove that Joe must have been speeding?

AROC $= \frac{200 \text{ km}}{2 \text{ hours}} = 100 \text{ km/h}$

May 29-1:11 PM

Hyperbolic Functions

Picture cables, like power lines, which hang freely.

$y = \frac{e^x + e^{-x}}{2}$

Freely hanging cables such as this actually hang in curves called hyperbolic cosine curves.

The hyperbolic cosine function is defined as follows: $\cosh x = \frac{e^x + e^{-x}}{2}$

While this is more of an exponential function, it is given the name cosh x because its derivatives are very similar to those of the trig functions.

May 20-6:46 PM

$$\sinh x = \frac{e^x - e^{-x}}{2} \qquad \cosh x = \frac{e^x + e^{-x}}{2}$$

$$\tanh x = \frac{\sinh x}{\cosh x} \qquad \coth x = \frac{\cosh x}{\sinh x} = \frac{1}{\tanh x}$$

$$\operatorname{sech} x = \frac{1}{\cosh x} \qquad \operatorname{csch} x = \frac{1}{\sinh x}$$

$y = \frac{e^x - e^{-x}}{2}$

$y = \frac{1}{2}(e^x - e^{-x})$

$y' = \frac{1}{2}(e^x - e^{-x}(-1))$

$y' = \frac{1}{2}(e^x + e^{-x})$

e^x

$e^{2x} = e^{2x}(2)$

2

May 20-6:55 PM

Derivatives of the Hyperbolic Trig. Functions

① $\frac{d}{dx}(\sinh x) = \cosh x$ sinch

② $\frac{d}{dx}(\cosh x) = \sinh x$ cosh

③ $\frac{d}{dx}(\tanh x) = \operatorname{sech}^2 x$ tanch

$\frac{d}{dx}(\coth x) = -\operatorname{csch}^2 x$

$\frac{d}{dx}(\operatorname{sech} x) = -\operatorname{sech} x \tanh x$

$\frac{d}{dx}(\operatorname{csch} x) = -\operatorname{csch} x \coth x$

$\frac{d}{dx}\sin x = \cos x \qquad \frac{d}{dx}\csc x = -\csc x \cot x$

$\frac{d}{dx}\cos x = -\sin x \qquad \frac{d}{dx}\sec x = \sec x \tan x$

$\frac{d}{dx}\tan x = \sec^2 x \qquad \frac{d}{dx}\cot x = -\csc^2 x$

May 20-6:57 PM

$y = e^{\cosh(5x)}$

$y' = e^{\cosh(5x)} \sinh(5x)\ 5$

May 21-2:42 PM

Calculate the following derivative:

$$y = e^{\sinh(4x^2)}$$

$y' = e^{\sinh(4x^2)} \cosh(4x^2)\,(8x)$

Jun 7-3:03 PM

Ex: Determine the derivative of the following function:

$$y = \tanh\sqrt{1+t^2}$$

$y' =$

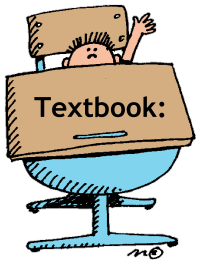
$y = \tanh(1+t^2)^{1/2}$

$y' = \operatorname{sech}^2(1+t^2)^{1/2}\ \frac{1}{2}(1+t^2)^{-1/2}(2t)$

May 20-6:59 PM

$y = x^2 + C$

$y' = 2x$



Green Book

p. 617 #13-24 (not 23...we did it already)

Jan 13-9:38 PM

Attachments

- 2.1_74_AP.html
- 2.1_74_AP.swf
- 2.1_74_AP.html